Particle Swarm Optimization

①

Ex To optimise the function $f(x) = x_1^2 + x_2^2$

Here $-5 \le x_1, x_2 \le 5$

Using PSO minimize the function $f(x)$

Sol^n let we generate 5 swarm using uniform distribution. Here possition vector X and velocity vector V is given below.

$$X = \begin{array}{c} \begin{bmatrix} 2.7045, & 4.8030 \\ 4.5974, & 2.8793 \\ 1.8710, & 4.0528 \\ 1.6400, & 1.3202 \\ 3.3392, & 0.9963 \end{bmatrix} \end{array} \begin{array}{l} - x_1 \\ - x_2 \\ - x_3 \\ - x_4 \\ - x_5 \end{array} \quad \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ x_{31} & x_{32} \\ x_{41} & x_{42} \\ x_{51} & x_{52} \end{bmatrix}$$

The velocity vector is generated uniformly in range [0, 1]

$$V = \begin{bmatrix} 0.4752 & 0.6987 \\ 0.4141 & 0.4020 \\ 0.7797 & 0.9433 \\ 0.6183 & 0.4749 \\ 0.2530 & 0.9398 \end{bmatrix} \begin{array}{l} - v_1 \\ - v_2 \\ - v_3 \\ - v_4 \\ - v_5 \end{array} \quad \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \\ v_{31} & v_{32} \\ v_{41} & v_{42} \\ v_{51} & v_{52} \end{bmatrix}$$

By using the initial fitness vector the fitness value of each swarm particle is

for
$$x_1 = f(x_1) = (2.7045)^2 + (4.8030)^2 = 30.3831$$
$$x_2 = f(x_2) = (4.5974)^2 + (2.8793)^2 = 29.4265$$
$$x_3 = f(x_3) = (1.8710)^2 + (4.0528)^2 = 19.9258$$
$$x_4 = f(x_4) = (0.6183)^2 + (0.4749)^2 = 4.4325$$
$$x_5 = f(x_5) = (0.2530)^2 + (0.9398)^2 = 12.1429$$

(2)

Here the minimum value is 4.4325, i.e for the swarm x_4, which is the best soln out (0th (itteraction) at this time.

So 4.4325 is the gbest value of partice x_4.

As this is the 0th iteration, so each particle current position is also the pbest possition.

Now we have to update the possition vector and velocity vector for iteraction - 1 by using the dynamic equation as below.

$$v_i^{t+1} = \underbrace{v_i^t}_{\text{inertia}} + \underbrace{c_1 r_1^t (pbest_i^t - P_i^t)}_{\text{personal influence}} + \underbrace{c_2 r_2^t (gbest^t - P_i^t)}_{\text{social influence}}$$

The position vector will be

$$P_i^{t+1} = P_i^t + v_i^{t+1}$$

<u>Here</u> $c_1 = c_2 = 2 \quad (0 \le c_1, c_2 \le 2)$

$r_1 = 0.34$

$r_2 = 0.86$ $\quad \{ 0 \le r_1, r_2 \le 1 \}$

2

iter-1 (velocity of v_1^1 3

$v_{11}^1 = v_{11}^0 + c_1 r_1 (Pbest_1^0 - x_{11}^0) + c_2 r_2 (gbest_1^0 - x_{11}^0)$

$= 0.475 + 2 \times 0.34 (2.7045 - 2.7045) + 2 * 0.86 (1.6400 - 2.7045)$

$= -1.35574$

Possition of x_1^1

$\Rightarrow x_{11}^1 = x_{11}^0 + v_{11}^1$

$= 2.7045 + (-1.35574)$

$= 1.34876$

Now, we have to check the condition that the updated possition value must be within the search space i.e (-5 to 5)

Here 1.34876 is within (-5 to 5)

So it can be acceptable.

for 2nd component (x_{12})

$v_{12}^1 = v_{12}^0 + c_1 r_1 (Pbest_{12}^0 - x_{12}^0) + c_2 r_2 (gbest_{12} - x_{12}^0)$

$= 0.6987 + 2 * 0.47 (4.8030 - 4.8030) + 2 * 0.91 (1.3202 - 4.8030)$

$= -5.351696$

$x_{12}^1 = x_{12}^0 + v_{12}^0 = 4.8030 + (-5.351696)$

$= -0.5486$.

So -0.548 also lies in the range (-5, 5)

4

So it can be acceptable,

So the first particle ~~as~~ x_1 after the 1st iteration the position becomes

$$x_1 = (1.34876, -0.5486)$$

Similarly for second particle x_2

$$\boxed{x_2 = (-0.0732, 3.0942)}$$

for 3rd particle

$$v_{31}' = v_{31}^0 + c_1 r_1 (Pbest_{31} - x_{31}^0) + c_2 r_2 (gbest_1^0 - x_{31}^0)$$

$$= 0.7797 + 2 * 0.98 (1.8710 - 1.8710) + 2 * 0.86 (1.6400 - 1.8710)$$

$$= 0.3824$$

$$x_{31}' = 1.8710 + 0.3824 = 2.2534$$

$$x_{32}' = 3.1379$$

So $\boxed{x_3 = (0.3824, 3.1379)}$

for 4th particle

$$\boxed{x_4^2 = (2.2583, 1.7951)}$$

for 5

So t

vect

$X^1 =$

V

Ma

for 5th particle (5)

$$\boxed{x_5 = (2.2668,\ 2.0009)}$$

So the position vector ~~after~~ and velocity vector after 1st iteration will be

$$X^1 = \begin{bmatrix} 1.3487, & -0.5486 \\ -0.0752, & 3.0942 \\ 2.2534, & 3.1379 \\ 1.6400, & 1.3202 \\ 2.2668, & 2.0009 \end{bmatrix} \quad \text{and}$$

$$V^1 = \begin{bmatrix} -1.3557, & -5.351696 \\ -4.6726, & 0.2149 \\ 0.3874, & -0.9149 \\ 0.6183, & 0.4749 \\ -1.0724, & 1.0046 \end{bmatrix}$$

Now the particle fitness value using the function $(f(x) = x_1^2 + x^2)$ for X^1 will be

$x_1 = \cancel{f(x_1)} = (1.3487)^2 + (-0.5486)^2 = 2.1200$

$x_2 = f(x_2) = (-0.0752)^2 + (3.0942)^2 = 9.5797$

$= 14.9242$

$x_3 = \quad = 8.3223$

$x_4 = \quad = 9.1420$

$x_5 =$

Now the pbest and gbest value will be change (5)

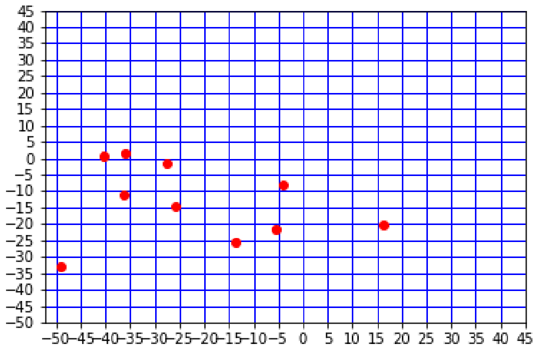
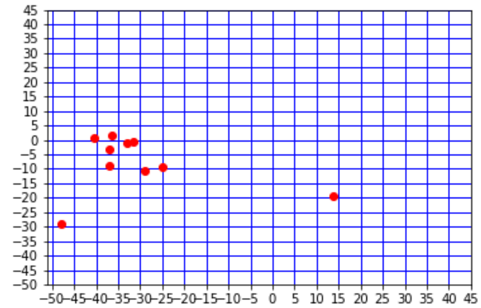
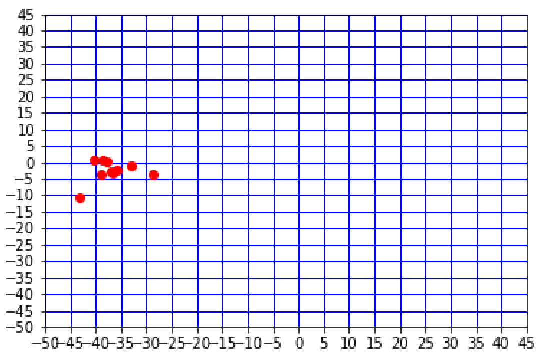
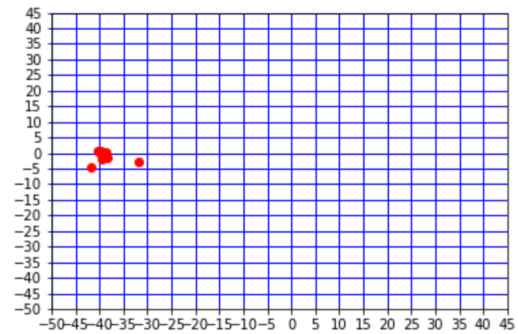
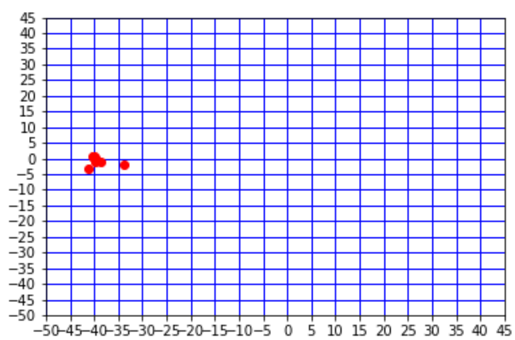
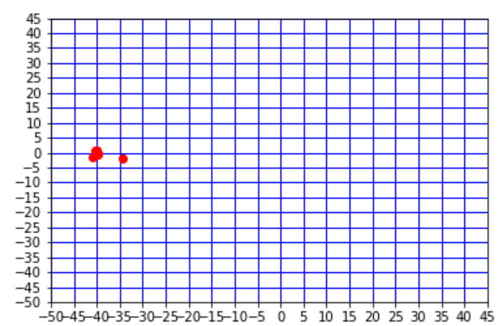
In 0th iteration the fitness value of each particle is

$$\begin{array}{l} x_1 - \\ x_2 - \\ x_3 - \\ x_4 - \\ x_5 - \end{array} \begin{bmatrix} 30.3831 \\ 29.4265 \\ 19.4258 \\ \boxed{4.4325} \\ 12.1429 \end{bmatrix} \text{— gbest } (x_4 - \text{particle})$$

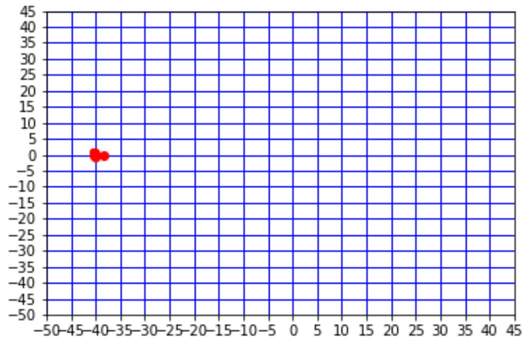
for 1st iteration the fitness value of each particle is

$$\begin{array}{l} x_1 - \\ x_2 - \\ x_3 - \\ x_4 - \\ x_5 - \end{array} \begin{bmatrix} \boxed{2.1200} \\ 9.5797 \\ 14.9242 \\ 8.3223 \\ 9.1420 \end{bmatrix} \text{— gbest } (x_1 - \text{particle})$$

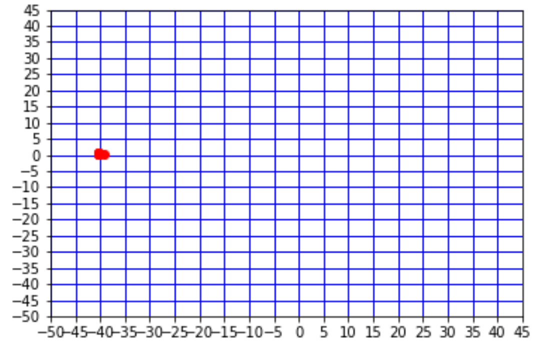
Similarly, we have to update the position vector of each particle unless all are converge to one point / to reach a stopping criteria.

Iteration 1:	Iteration 2:
	
Iteration 3:	**Iteration 4:**
	
Iteration 5:	**Iteration 6:**
	

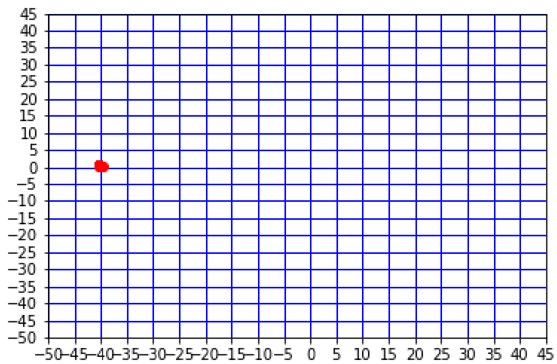
Iteration 7:



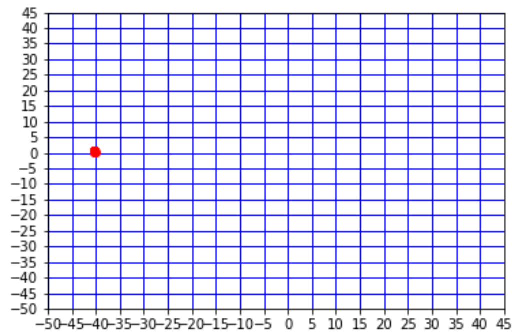
Iteration 8:



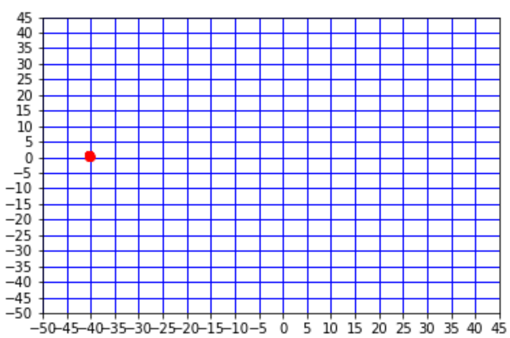
Iteration 9:



Iteration 10:



Iteration 11:



Iteration 12: